Peer-to-Peer Product Sharing: Implications for Ownership, Usage and Social Welfare in the Sharing Economy

Saif Benjaafar[*], Guangwen Kong[†], Xiang Li[‡]

Department of Industrial and Systems Engineering, University of Minnesota

Costas Courcoubetis[§]

Engineering and Systems Design, Singapore University of Technology and Design

Abstract

We describe an equilibrium model of peer-to-peer product sharing, or collaborative consumption, where individuals with varying usage levels make decisions about whether or not to own. Owners are able to generate income from renting their products to non-owners while non-owners are able to access these products through renting on as needed basis. We characterize equilibrium outcomes, including ownership and usage levels, consumer surplus, and social welfare. We compare each outcome in systems with and without collaborative consumption and examine the impact of various problem parameters including rental price, platform's commission fee, cost of ownership, owner's moral hazard cost, and renter's inconvenience cost. Our findings indicate that, depending on the rental price, collaborative consumption can result in either lower or higher ownership and usage levels, with higher ownership and usage levels more likely when the cost of ownership is high. We show that consumers always benefit from collaborative consumption, with individuals who, in the absence of collaborative consumption, are indifferent between owning and not owning benefitting the most. We also show that the platform's profit is not monotonic in the cost of ownership, implying that a platform is least profitable when the cost of ownership is either very high or very low.

Keywords: sharing economy; collaborative consumption; two-sided markets; social welfare; sustainability

[*]saif@umn.edu
[†]gkong@umn.edu
[‡]lixx1315@umn.edu
[§]costas@sutd.edu.sg

1 Introduction

We are witnessing, across a wide range of domains, a shift away from the exclusive ownership and consumption of resources to one of shared use and consumption. This shift is taking advantage of innovative new ways of peer-to-peer sharing that are voluntary and enabled by internet-based exchange markets and mediation platforms. Value is derived from the fact that many resources are acquired to satisfy infrequent demand but are otherwise poorly utilized (for example, the average car in the US is used less than 5 percent). Several successful businesses in the US and elsewhere, such as AirbnB for homes, RelayRides for cars, LiquidSpace for office space, JustPark for parking, and StyleLend for designer clothing, provide a proof of concept and evidence for the viability of peer-to-peer product sharing or collaborative consumption (the term we use in the rest of the paper). These businesses and others allow owners to rent on a short-term basis poorly utilized assets and non-owners to access these assets through renting on an as-needed basis. Collectively, these businesses and other manifestations of the collaborative consumption of products and services are giving rise to what is becoming known as the sharing economy[1].

The peer-to-peer sharing of products is not a new concept. However, recent technological advances in several areas have made it more feasible by lowering the associated search and transactions costs. These advances include the development of online marketplaces, mobile devices and platforms, electronic payments, and two-way reputation systems whereby users rate providers and providers rate users. Other drivers behind the rise of collaborative consumption are societal and include increased population density in metropolitan areas around the world, increased concern about the environment (collaborative consumption is viewed as a more sustainable alternative to traditional modes of consumption), and increased desire for community and altruism among the young and educated.

Collaborative consumption has the potential of increasing access while reducing investments in resources and infrastructure. In turn, this could have the twin benefit of improving consumer welfare (individuals who may not otherwise afford a product now have an opportunity to use it) while

[1]The term sharing economy has been used to refer to businesses that enable the foregoing of ownership in favor of "on-demand" access. In several cases, this involves a single entity that owns the physical assets (e.g., Zipcar for short term car rentals). It also encompasses the peer-to-peer provisioning of services (e.g., Uber for transportation services, TaskRabbit for errands, and Postmates for small deliveries). For further discussion and additional examples, see Botsman and Rogers [2010], Malhotra and Alstyne [2014], Cusumano [2014], and Chase [2015].

reducing societal costs (externalities, such as pollution that may be associated with the production, distribution, use, and disposal of the product). It also has the potential of providing a source of net income for owners by monetizing poorly utilized assets, which are in some cases also expensive and rapidly depreciating. Take cars for example. The availability of a sharing option could lead some to forego car ownership in favor of on-demand access. In turn, this could result in a corresponding reduction in congestion and emissions and, eventually, in reduced investments in roads and parking infrastructure. However, increased collaborative consumption may have other consequences, some of which may be undesirable. For example, greater access to cars could increase car usage and, therefore, lead to more congestion and pollution if it is not accompanied by a sufficient reduction in the numbers of cars[2]. This could occur if sharing leads to speculative investments in cars and price inflation, or yet if it affects the usage, availability and pricing of other modes of public transport, such as taxis, buses, and trains.

Collaborative consumption raises several important questions. How does collaborative consumption affect ownership and usage of resources? Is it necessarily the case that collaborative consumption leads to lower ownership, lower usage, or both (and therefore to improved sustainability)? If not, what conditions would favor lower ownership, lower usage, or both? Who benefits the most from collaborative consumption among owners, and renters? To what extent would a profit-maximizing platform, through its choice of rental prices, improve social welfare? To what extent do frictions, such as moral hazard (additional wear and tear renters place on rented resources) and inconvenience experienced by renters affect platform profit and social welfare?

In this paper, we address these and other related questions. We describe an equilibrium model of peer-to-peer product sharing, where individuals with varying usage levels make decisions about whether or not own. In the presence of collaborative consumption, owners are able to generate income from renting their products to non-owners while non-owners are able to access these products through renting. The matching of owners and renters is facilitated by a platform, which sets the rental price and charges a commission fee. Because, supply and demand can fluctuate over the short run, we allow for the possibility that an owner may not always be able to find a renter when

[2]A recent article in the New York Times (Editorial [2015]) notes that "The average daytime speed of cars in Manhattan's business districts has fallen to just under 8 miles per hour this year, from about 9.15 miles per hour in 2009. City officials say that car services like Uber and Lyft are partly to blame. So Mayor Bill de Blasio is proposing to cap their growth."

she puts her product up for rent. Similarly, we allow for the possibility that a renter may not always be able to find a product to rent when he needs one. We refer to the uncertainty regarding the availability of renters and products as matching friction and describe a model for this uncertainty. We also account for the moral hazard cost incurred by owners due to the additional wear and tear that a renter places on a rented product and for the inconvenience cost experienced by renters for using a product that is not their own.

For a given price and a commission fee, we characterize equilibrium ownership and usage levels, consumer surplus, and social welfare. We compare each in systems with and without collaborative consumption and examine the impact of various problem parameters including price, commission fee, cost of ownership, moral hazard cost, and inconvenience cost. We also characterize equilibrium outcomes when the platform decides on the rental price to maximize its own profit. We compare the resulting social welfare to that realized in the absence of collaborative consumption and to that obtained under a social welfare-maximizing platform. Our main findings include the following:

- Depending on the rental price, we show that collaborative consumption can result in either higher or lower ownership. In particular, we show that when the rental price is sufficiently high (above a well-specified threshold), collaborative consumption leads to higher ownership. We show that this threshold is decreasing in the cost of ownership. That is, collaborative consumption is more likely to lead to more ownership when the cost of ownership is high (this is because collaborative consumption allows individuals to offset the high ownership cost and pulls in a segment of the population that may not otherwise choose to own).

- Similarly, we show that collaborative consumption can lead to either higher or lower usage, with usage being higher when price is sufficiently high. Thus, it is possible for collaborative consumption to result in both higher ownership and higher usage (it is also possible for ownership to be lower but usage to be higher and for both ownership and usage to be lower).

- We show that consumers always benefit from collaborative consumption, with individuals who, in the absence of collaborative consumption, are indifferent between owning and not owning benefitting the most. This is because among non-owners those with the most usage (and therefore end up renting the most) benefit the most from collaborative consumption. Similarly, among owners, those with the least usage (and therefore end up earning the most rental income) benefit the most.

- We show that the platform's profit is not monotonic in the cost of ownership, first increasing and then decreasing, implying that a platform is least profitable when the cost of ownership is either very high or very low (those two extremes lead to scenarios with either mostly renters and few owners or mostly owners and few renters). The platform is most profitable when owners and renters are sufficiently balanced.

- We observe that the platform's profit is also not monotonic in the cost of moral hazard, implying that a platform may not have an incentive to completely eliminate moral hazard. This is because the platform can leverage moral hazard to induce desirable ownership levels without resorting to extreme pricing, which can be detrimental to its revenue.

The rest of the paper is organized as follows. In Section 2, we provide a review of related literature. In Section 3, we describe our model. In Section 4, we provide an analysis of the equilibrium. In Section 5, we consider the platform's problem. In Section 6, we offer concluding comments.

2 Related Literature

Our work is related to the literature on two-sided markets (see for example Rochet and Tirole [2006]; Weyl [2010]; Hagiu and Wright [2015]) and network externalities (Liebowitz and Margolis [1994]; Katz and Shapiro [1985]; Ambrus and Argenziano [2004]). Examples of two-sided markets include video game platforms which need to attract both game developers to design games and game players to use the video game platform; social media which bring together members and advertisers; and operating systems for computers and smart phones, which connect users and application developers. A common feature of two-sided markets is that the utility of individuals on each side of the market increases with the size of the other side of the market. As a result, it can be beneficial for the platform to heavily subsidize one side of the market (e.g., social media sites are typically free to members). Collaborative consumption is different from two-sided markets in several ways, the most important of which is that the two sides are not distinct. In collaborative consumption, being either an owner or a renter is a decision that users of the platform make, with more owners implying fewer renters (and vice-versa). Therefore, heavily subsidizing one side of the market may not necessarily be desirable as it can create an imbalance in the supply and demand for the shared resource.

There is extensive literature that deals with the social sharing (piracy) of information goods (see for example Novos and Waldman [1984]; Johnson and Waldman [2003]; and Besen and Kirby [1989]). An important result from this literature is that allowing for some piracy can actually benefit the firm that supplies the information good (as well as maximize social welfare). This is because the firm can, with its choice of prices, strategically target sharing groups rather than individuals (e.g., Bakos et al. [1999]; Galbreth et al. [2012]). The firm can also benefit from positive network externalities (e.g., Shy and Thisse [1999]; Varian [2005]) and from reduced price competition as price sensitive consumers copy instead of buy (e.g., Jain [2008]). The sharing of physical goods, as we consider in this research, is substantially different from the illegal sharing of information goods. For example, physical goods cannot be costlessly duplicated and the owner of a physical good must forego consumption while the good is being rented. As a result, the frequency of usage for each individual is not as relevant as it is in the collaborative consumption of physical goods.

Our work is also related to the literature on secondary markets for durable goods. Used products with inferior quality or older vintage sold on a secondary market can compete with a firm's new products, reducing the demand for such products. The presence of a secondary market can, on the other hand, enhance the perceived value of such products (and consequently demand) as customers account for the products' resale value when making buying decisions. Hence, firms must consider these counteracting effects in making product pricing and durability decisions. Examples from this literature include Waldman [1993]; Waldman [1996]; Waldman [1997]; Fudenberg and Tirole [1998]; Chevalier and Goolsbee [2009]; and Chen et al. [2013], and the references therein. A review of this literature can be found in Waldman [2003].

Markets with collaborative consumption involving peer-to-peer short-term rentals, such as the one we study in this paper, are different from those with a secondary market for used goods. The latter involves the permanent transfer of ownership from the seller to the buyer and does not have the feature of joint consumption among owners and renters. More significantly, a secondary market for used goods does not have the feature of uncertain demand and supply present in the setting we study. Nevertheless, we show, that ownership can increase with collaborative consumption if the rental income is sufficiently high.

There is a small but growing number of papers that deal with peer-to-peer marketplaces with collaborative consumption features. For example, Fradkin et al. [2015] studies sources of inefficiency in matching buyers and suppliers in online market places. Using a counterfactual study, they show how changes to the ranking algorithm of Airbnb can improve the rate at which buyers are successfully matched with suppliers. Zervas et al. [2015] examine the relationship between Airbnb supply and hotel room revenue in Texas and find that an increase in Airbnb supply has only a modest negative impact on hotel revenue. Cullen and Farronato [2014] describe a model of peer-to-peer labor marketplaces. They calibrate the model using data from TaskRabbit and find that supply is highly elastic, with increases in demand matched by increases in supply per worker with little or no impact on price.

Papers that are closest in spirit to ours are Fraiberger and Sundararajan [2015] and Jiang and Tian [2015]. Fraiberger and Sundararajan [2015] describe a dynamic programing model where individuals make decisions in each period regarding whether to purchase a new car, purchase a used a car, or not purchase anything. They model matching friction, as we do, but assume that the renter-owner matching probabilities are exogenously specified and not affected by the ratio of owners to renters (in our case, we allow for these to depend on the ratio of owners to renters which turns out to be critical in the decisions of individuals on whether to own or rent). They assume that the rental income, determined by a single market-clearing price, is transferred entirely from renters to owners (they do not model the platform explicitly and assume the rental price is determined by a single market clearing price). They provide numerical results that show that collaborative consumption leads to a reduction in new and used car ownership, an increase in the fraction of the population who do not own, and an increase in the usage intensity per vehicle. In this paper, we provide analytical results regarding ownership and usage, and provide conditions under which either one increases.

Jiang and Tian [2015] describe a two-period model, where individuals first decide on whether or not to own a product. This is followed by owners deciding in each period on whether to use the product themselves or rent it. They assume that demand always matches supply through a market clearing price and do not consider the possibility of a mismatch, because of matching frictions, between supply and demand. They take the perspective of the product manufacturer and study the manufacturer's pricing and quality decisions in view of the product-sharing market, which is different from our focus on the platform and on outcomes regarding ownership and usage. They show that the moral hazard cost (the additional wear and tear renters place on the product) and the platform's commission can have a non-monotonic effect on the profits of the original manufacturer of the product, the surplus of consumers, and social welfare.

An important application of collaborative consumption is shared mobility. Although the literature on peer-to-peer sharing is limited, there is literature that studies shared mobility that does not involve individual ownership. Such systems typically consist of a service provider who owns the vehicles, such as cars or bikes, and consumers who rent these vehicles from the service provider. A common feature of these services is their flexibility (e.g., rental periods can be in small time increments and the vehicles can be accessed from/returned to multiple locations). A stream within this literature focuses on the operation and logistics of these sharing systems (see for example, Schuijbroek et al. [2013]; Raviv and Kolka [2013]; Shu et al. [2013]). Another stream examines the sustainability of vehicle sharing relative to traditional ownership (see for example Cervero et al. [2007], Lane [2005], Cervero et al. [2007], Martin et al. [2010], and Martin and Shaheen [2011]). Some of the empirical findings from this literature indicate that car sharing increases emissions by expanding access to cars. Our work is different from the above-mentioned literature in that there is not a single entity that owns all the vehicles and with owners being simultaneously consumers and service providers. However, we do provide results regarding ownership and usage and conditions under which either one increases under collaborative consumption.

3 Model Description

In this section, we describe our model of collaborative consumption. We reference the case of car sharing. However, the model applies more broadly to the collaborative consumption of other products. We consider a population of individuals who are heterogeneous in their product usage, with their type characterized by their usage level ξ. We assume that the utility derived by an individual with type ξ, $u(\xi)$ is concave increasing in ξ. Without loss of generality, we normalize the usage level to $[0, 1]$, where $\xi = 0$ corresponds to no usage at all and $\xi = 1$ to full usage. We let $f(\xi)$ denote the density function of the usage distribution in the population.

We assume products are homogeneous in their features, quality, and cost of ownership. In the absence of collaborative consumption, each individual makes a decision about whether or not to own. In the presence of collaborative consumption, each individual decides on whether to own, rent from others who own, or neither. Owners incur the fixed cost of ownership but can now generate income by renting their products to others who choose not to own. Renters pay the rental fee but avoid the fixed cost of ownership.

We let p denote the rental price per unit of usage that renters pay (a uniform price is consistent with observed practices by certain peer-to-peer platforms when the goods are homogenous). This rental price may be set by a third party platform (an entity that may be motivated by profit, total social welfare, or some other concern). The platform extracts a commission from successful transactions, which we denote by γ, where $0 \leq \gamma < 1$, so that the rental income seen by the owner per unit of usage is $(1-\gamma)p$. We let α, where $0 \leq \alpha \leq 1$ denote the probability in equilibrium that an owner, whenever she puts her product up for rent, is successful in finding a renter. Similarly, we denote by β, where $0 \leq \beta \leq 1$, the probability that a renter, whenever he decides to rent, is successful in finding an available product (the probabilities α and β are determined endogenously in equilibrium). A renter resorts to his outside option (e.g., pubic transport in the case of cars) whenever he is not successful in finding a product to rent. The owner incurs a fixed cost of ownership, denoted by c. Whenever the product is rented, the owner incurs an additional cost, denoted by w, due to extra wear and tear the renter places on the product (a moral hazard the owner faces because of the renter's potential negligence and mishandling of the product). Renters, on the other hand, incur an inconvenience cost, denoted by d (in addition to paying the rental fee), from using someone else's product and not their own. Without loss of generality, we assume that $c, d, p, w \in [0, 1]$.

We assume that an owner would always put her product out for rent when she is not using it. In other words, usage corresponds to the fraction of time an owner would like to have access to her product, regardless of whether or not she is actually using it. An owner has always a priority in accessing her product. Hence her usage can always be fulfilled. We also assume that a renter always prefers renting to the outside option. Otherwise, rentals would never take place as the outside option is assumed to be always available. There are of course settings where an individual would use a mix of options (e.g., different transportation methods). In that case, ξ corresponds to the portion of usage that an individual prefers to fulfill using the product (e.g., a car and not public transport).

The *payoff* of an owner with usage level ξ can now be expressed as[3]

$$\pi_o(\xi) = u(\xi) + (1-\xi)\alpha[(1-\gamma)p - w] - c, \tag{1}$$

while the payoff of a renter as

$$\pi_r(\xi) = u(\beta\xi) - (p+d)\beta\xi. \tag{2}$$

The payoff of an owner has three terms: the utility derived from usage, the income derived from renting (net of the wear and tear cost), and the cost of ownership. The income from renting is realized only when the owner is able to find a renter. The payoff of a renter is the difference between the utility derived from renting and the cost of renting (the sum of rental price and inconvenience cost). This payoff is realized only for the fraction of usage that can be satisfied through rental. Without loss of generality, the value of the outside option (e.g., using public transport) is normalized to zero[4].

An individual with type ξ would participate in collaborative consumption as an *owner* if the following conditions are satisfied

$$\pi_o(\xi) \geq \pi_r(\xi) \text{ and } \pi_o(\xi) \geq 0.$$

The first constraint is an incentive compatibility constraint that ensures that an individual with type ξ prefers to be an owner rather than be a renter. The second constraint is a participation constraint that ensures the individual participates in collaborative consumption. Similarly, an individual with type ξ would participate in collaborative consumption as a *renter* if the following conditions are satisfied

$$\pi_r(\xi) \geq \pi_o(\xi) \text{ and } \pi_r(\xi) \geq 0.$$

[3]We assume usage is exogenously determined and unaffected by the presence of collaborative consumption (i.e., the usage of each individual is mostly inflexible and must be satisfied through either renting or owning).

[4]Suppose the utility derived from using an outside option is o per unit time. Then, the renter's payoff becomes $\pi_r(\xi) = u(\beta\xi) - (p+d)\beta\xi + o(1-\beta)\xi$. Letting $v(\xi) = u(\xi) - o\xi$, π_o can be simplified to $\pi_o(\xi) = v(\xi) + (1-\xi)\alpha[(1-\gamma)p - w] - c + o\xi$, and π_r to $\pi_r(\xi) = v(\beta\xi) - (p+d)\beta\xi + o\xi$. An individual's decision, of being an owner or renter, does not change if $o\xi$ is eliminated from both equations. This leads to (1) and (2) with $v(\xi)$ in place of $u(\xi)$.

If $\pi_o(\xi) - \pi_r(\xi)$ is monotonically increasing in ξ and $u(\theta) \geq (p+d)\theta$ for $\theta \in [0,1]$, collaborative consumption would take place if there exists $\theta \in (0,1)$ such that

$$\pi_o(\theta) = \pi_r(\theta). \tag{3}$$

The parameter θ would then segment the population into owners and renters, where individuals with $\xi > \theta$ are owners and individuals with $\xi < \theta$ are renters (an individual with $\xi = \theta$ is indifferent between owning and renting). We refer to $\omega = \int_{[\theta,1]} f(\xi)d\xi$, the fraction of owners in the population, as the *ownership level* or simply *ownership*.

In the absence of collaborative consumption, an individual would own a car if $u(\xi) \geq c$ and would not otherwise. Let $\hat{\theta}$ denote the solution to $u(\xi) = c$. Then, the fraction of the population that corresponds to owners (ownership) is $\hat{\omega} = \int_{[\hat{\theta},1]} f(\xi)d\xi$.

3.1 Matching Supply with Demand

In the presence of collaborative consumption, let $D(\theta)$ denote the aggregate demand (for car rentals) generated by renters and $S(\theta)$ the aggregate supply generated by owners, for given θ. Then,

$$D(\theta) = \int_{[0,\theta)} \xi f(\xi)d\xi,$$

and

$$S(\theta) = \int_{[\theta,1]} (1-\xi)f(\xi)d\xi.$$

In addition, let $q(\theta) = \int_{[\theta,1]} \xi f(\xi)d\xi + \beta \int_{[0,\theta]} \xi f(\xi)d\xi$ denote the total usage, where the first term is usage due to owners and the second term is usage due to renters (note the second term is modulated by β).

Let

$$\rho(\theta) = \frac{D(\theta)}{S(\theta)}. \tag{4}$$

Then $\rho(\theta)$ can be viewed as a measure of the relative demand for the available cars. A higher $\rho(\theta)$ indicates that it is more likely for an owner to rent her car, implying a higher owner surplus. However, a higher $\rho(\theta)$ also indicates that a renter is less likely to find an available car, implying a lower renter surplus. Hence, with collaborative consumption, there is ongoing tension between having too many renters (increases α but decreases β) and too many owners (decreases α but increases β). This tension is resolved in equilibrium via θ, which balances the payoff of owners and renters and determines the fraction of each in the population.

For a given θ, the amount of demand from renters that is fulfilled must equal the amount of supply from owners that is matched with renters. In other words, the following fundamental relationship must be satisfied

$$\alpha S(\theta) = \beta D(\theta). \tag{5}$$

The parameters α and β, along with θ, are determined endogenously in equilibrium.

In constructing a model for α and β, the following are desirable properties: (*i*) α (β) increases in (deceases) in θ; (*ii*) α approaches 1 (0) when θ approaches 1 (0); (*iii*) β approaches 1 (0) when θ approaches 0 (1), and (*iv*) higher α implies a lower β. A plausible model for α and β is one that arises naturally from a *multi-server loss queueing system* approximation[5]. In such a system, $1 - \beta$ would correspond to the blocking probability (the probability that a request for rental finds all products rented out, or, in queueing parlance, a request finds all servers busy) and α corresponds to the probability that an available product (server) is rented (busy). Assuming the arrival of rental requests can be approximated by a Poisson process, we can approximate α as follows (see for example Sobel [1980])

$$\alpha = \frac{\rho(\theta)}{1 + \rho(\theta)}. \tag{6}$$

Applying Little's law leads to

$$\beta = \frac{1}{1 + \rho(\theta)}. \tag{7}$$

[5]In the corresponding queueing system, the arrival process is that of rental requests. If we let m denote the mean rental time per each rental, the arrival rate (in terms of rental requests per unit time) is given by $\lambda(\theta) = D(\theta)/m$. For example, if the aggregate demand for renting per unit time is $D(\theta) = 1000$ hours per unit time and the average rental period is $m = 5$ hours, then the arrival rate of rental requests is $D(\theta)/m = 200$ requests per unit time. If we approximate the number (in equilibrium) of products available for rent per unit time by a constant, say $K(\theta)$, then the service capacity in the system (the number of rental requests that can be fulfilled per unit time) is given by $C(\theta) = \frac{S(\theta)}{m}$, where $S(\theta)$ is the aggregate amount of time products are available for rent. For example, if $S(\theta) = 2000$ hours and $m = 5$ hours, then $C(\theta) = 400$ rental requests per unit time. Thus, we can express the workload (the ratio of the arrival rate to service capacity) as $\rho(\theta) = \frac{\lambda(\theta)}{C(\theta)} = \frac{D(\theta)}{S(\theta)}$. Note that we do not have to compute $K(\theta)$ explicitly as the approximation we use to estimate α and β depends only the workload $\rho(\theta)$.

As a result, we have $\alpha + \beta = 1$. Substituting the expression of $\rho(\theta)$ from (4) into (6) leads to

$$\alpha = \frac{D(\theta)}{D(\theta) + S(\theta)}, \tag{8}$$

and $\beta = \frac{S(\theta)}{D(\theta)+S(\theta)}$. Note that α and β, as defined, satisfy the demand-supply balance equation (5) and properties (i)-(iv) stated above. Note that the above expressions for α and β can also be obtained directly from the demand-supply balance equation (5) by setting $\beta = 1 - \alpha$ and solving for α (that is, these are the unique values of α and β satisfying (5) and $\alpha + \beta = 1$). The model for α and β specified in (6) and (7) is of course not unique in satisfying properties $(i) - (iv)$. However, we expect other plausible models that satisfy these properties to lead to results that are qualitatively similar to those we describe in the next two sections.

An equilibrium under collaborative consumption exists if there exists $(\theta, \alpha) \in (0, 1)^2$ that is solution to (3) and (8). When it exists, we denote this solution by (θ^*, α^*). Knowing the equilibrium allows us to answer important questions regarding car ownership, overall usage, and social welfare, among others.

4 Equilibrium Analysis

In this section, we focus on the case where the utility function has the linear form $u(\xi) = \xi$, and ξ is uniformly distributed in $[0, 1]$. The utility function has constant returns to scale, and the utility derived from each unit of usage is normalized to 1. It is straightforward to consider a utility function in general linear form and carry out similar analysis. We use a linear utility for ease of exposition and to allow for closed form expressions. Then the rental price must satisfy $\frac{w}{1-\gamma} \leq p \leq 1 - d$ since otherwise, the owners or renters will not share. We denote the set of admissible prices by

$$A = \{p | \frac{w}{1-\gamma} \leq p \leq 1 - d\}. \tag{9}$$

Letting θ denote the solution to $\pi_o(\xi) = \pi_r(\xi)$ leads to

$$\theta = \frac{c - ((1-\gamma)p - w)\alpha}{p + d + (1 - p - d)\alpha - ((1-\gamma)p - w)\alpha}. \tag{10}$$

Given θ, the aggregate demand under collaborative consumption is given by $D(\theta) = \frac{\theta^2}{2}$ and aggregate supply by $S(\theta) = \frac{(1-\theta)^2}{2}$. This leads to

$$\rho(\theta) = \frac{\theta^2}{(1-\theta)^2},$$

and by (6)

$$\alpha = \frac{\theta^2}{(1-\theta)^2 + \theta^2}. \tag{11}$$

An equilibrium exists if equations (10) and (11) admit a solution (θ^*, α^*) such that $(\theta^*, \alpha^*) \in (0,1)^2$. In the following theorem, we establish the existence and uniqueness of such an equilibrium. Let

$$\Omega = \{(p, \gamma, c, w, d) | c \in (0,1), \gamma \in [0,1), (w,d) \in [0,1]^2, p \in A\},$$

and Ω° be the interior of Ω.

Theorem 1. *A unique equilibrium (θ^*, α^*) exists for each $(p, \gamma, c, w, d) \in \Omega$.*

The existence of the equilibrium is guaranteed by the Intermediate Value Theorem. The uniqueness is due to the monotonicity of (10) and (11); see the Appendix for a proof of this and all subsequent results.

The following lemma describes how the equilibrium (θ^*, α^*) varies with the price p, commission γ, cost of ownership c, moral hazard cost w, and inconvenience cost d.

Lemma 2. *$(\theta^*, \alpha^*) : \Omega \to (0,1)^2$ is continuous on Ω, and continuously differentiable on Ω°. Moreover, $\frac{\partial \theta^*}{\partial p} < 0$, $\frac{\partial \alpha^*}{\partial p} < 0$, $\frac{\partial \theta^*}{\partial \gamma} > 0$, $\frac{\partial \alpha^*}{\partial \gamma} > 0$, $\frac{\partial \theta^*}{\partial c} > 0$, $\frac{\partial \alpha^*}{\partial c} > 0$, $\frac{\partial \theta^*}{\partial w} > 0$, $\frac{\partial \alpha^*}{\partial w} > 0$, $\frac{\partial \theta^*}{\partial d} < 0$, and $\frac{\partial \alpha^*}{\partial d} < 0$.*

Lemma 2 indicates that, in equilibrium, the population of renters θ^* increases with the cost of ownership c, the commission γ, and the wear and tear cost w, but decreases with the rental price p and the inconvenience cost d. Similarly, the probability that a car owner is successful in renting her car out α^*, as an increasing function of θ^*, increases with c, γ, and w, and decreases with p and d. These results are consistent with intuition.

In the presence of collaborative consumption, ownership in equilibrium, which we denote by ω^*, and total usage level, which we denote by q^*, are respectively given by

$$\omega^* = 1 - \theta^*, \tag{12}$$

and

$$q^* = \frac{1 - \alpha^* \theta^{*2}}{2}. \tag{13}$$

Proposition 3. *Ownership ω^* and usage q^* both (i) strictly decrease in the cost of ownership c, commission γ, and wear and tear cost w and (ii) strictly increase in rental price p and inconvenience cost d.*

Proposition 3 is a direct consequence of Lemma 2. While the monotonicity results in Proposition 3 are perhaps expected, it is not clear how ownership and usage under collaborative consumption compare to those under no collaborative consumption. In what follows, we provide comparisons between systems with and without collaborative consumption, and address the questions of whether or not collaborative consumption reduces car ownership and usage.

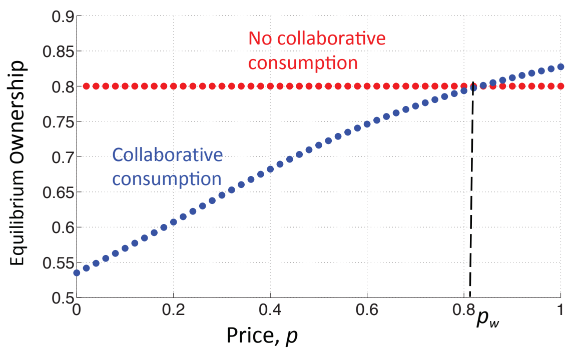
In the absence of collaborative consumption, ownership and usage, denoted respectively by $\hat{\omega}$ and $\hat{q}$, are given by

$$\hat{\omega} = 1 - c \text{ and } \hat{q} = \frac{1 - c^2}{2}. \tag{14}$$

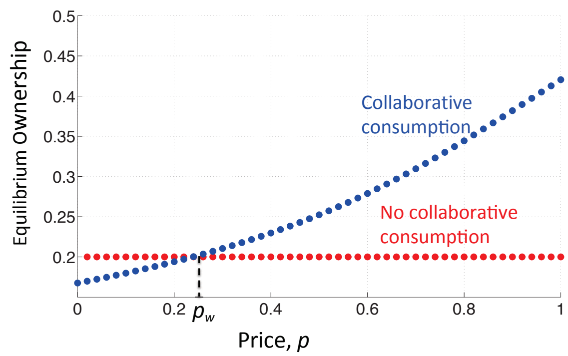
In the following proposition, we assume that $\frac{w}{1-\gamma} < 1-d$ so that the set of admissible prices consists of more than a single price.

Proposition 4. *Let $p_\omega = \frac{(1-d)(1-c)+wc}{1-\gamma c}$. Then, $\frac{w}{1-\gamma} < p_\omega < 1-d$, $\omega^* < \hat{\omega}$ if $p < p_\omega$, $\omega^* = \hat{\omega}$ if $p = p_\omega$, and $\omega^* > \hat{\omega}$ if $p > p_\omega$. Moreover, $\frac{\partial p_\omega}{\partial \gamma} > 0$, $\frac{\partial p_\omega}{\partial c} < 0$, $\frac{\partial p_\omega}{\partial w} > 0$, and $\frac{\partial p_\omega}{\partial d} < 0$.*

Proposition 4 shows that depending on the rental price p, collaborative consumption can result in either lower or higher ownership. In particular, when the rental price p is sufficiently high (above the threshold p_ω), collaborative consumption leads to higher ownership (e.g., more cars). Moreover, the threshold above which prices must be for this to occur is decreasing in the cost of ownership and renter's inconvenience, and increasing in the commission fee and wear and tear cost. The fact that p_ω is decreasing in c is perhaps surprising as it shows that collaborative consumption is more likely to lead to more ownership (and not less) when the cost of owning is high. Collaborative consumption in this case allows individuals to offset the high ownership cost and pulls in a segment of the population that may not otherwise choose to own. The reverse is of course also true. Collaborative consumption is more likely to lead to lower car ownership when the ownership cost is low. These effects are illustrated for an example system in Figure 1.



(a) $c = 0.2$, $\gamma = 0.2$, $w = 0$, $d = 0$



(b) $c = 0.8$, $\gamma = 0.2$, $w = 0$, $d = 0$

Figure 1: Impact of price on ownership

Similarly, usage can be either lower or higher with collaborative consumption than without it. In this case, there is again a price threshold p_q above which usage is higher with collaborative consumption, and below which usage is higher without collaborative consumption. When either w or d is sufficiently high, collaborative consumption always leads to higher usage. The result is formally stated in Proposition 5 and illustrated for an example case with $w = d = 0$ in Figure 2.

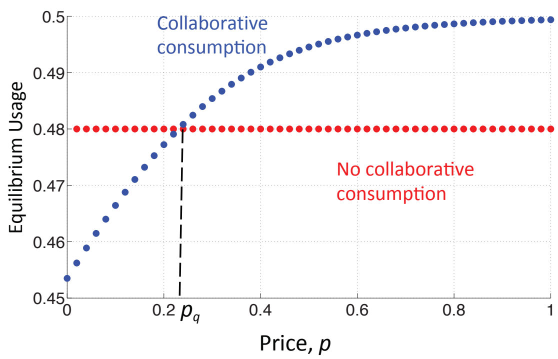
Proposition 5. *One of the following is true:*

(i) There exists $p_q \in (\frac{w}{1-\gamma}, 1-d)$ *such that* $q^* < \hat{q}$ *if* $p < p_q$, $q^* = \hat{q}$ *if* $p = p_q$, *and* $q^* > \hat{q}$ *if* $p > p_q$;

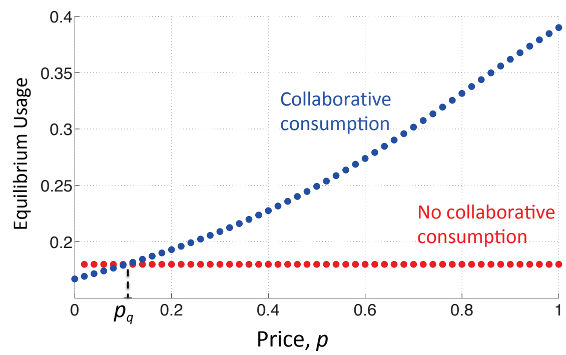
(ii) $q^* \geq \hat{q}$ *for all* $p \in [\frac{w}{1-\gamma}, 1-d]$.

Moreover, there exists $t \in (0, 1)$ *such that (i) is true if* $\frac{w}{1-\gamma} + d < t$, *and (ii) is true otherwise.*

Proposition 5 suggests that the opportunity to lower usage diminishes as either the moral hazard cost w or the inconvenience cost d increases. The effect of d is perhaps easy to understand as higher inconvenience cost reduces renters' payoffs, leading to higher ownership and higher usage (per proposition 3). The effect of w is on the other hand more subtle. As w increases, the minimum admissible price (for collaborative consumption to take place) also increases (recall that p must satisfy $p \geq \frac{w}{1-\gamma}$). The higher moral hazard cost reduces owners' payoffs. However, this could be made up for with the higher price. It turns out that the effect of price dominates the effect of moral hazard, such that, when the moral hazard is sufficiently high, collaborative consumption results in higher usage at any admissible price.



(a) $c = 0.2$, $\gamma = 0.2$, $w = 0$, $d = 0$



(b) $c = 0.8$, $\gamma = 0.2$, $w = 0$, $d = 0$

Figure 2: Impact of price on usage

Unlike p_ω, the price threshold p_q is not monotonic in c (see Figure 3). As c varies, p_q first increases then decreases. This can be explained as follows. For collaborative consumption to lead to lower usage, the ownership level must be sufficiently low (certainly lower than that without collaborative consumption) such that the decrease in owners' usage is greater than the increase in renters' usage. When the cost of ownership is low, the abundance of owners makes it easy for the renters to find available cars. As a result, the individuals who switch from being owners to being renters are able to fulfill most of their usage via renting (their usage is little to begin with.) Therefore, ownership level has to be much lower compared to that without collaborative consumption for there to be lower usage. This results in a low price threshold (recall that lower prices induce owners to become renters). When the cost of the ownership is high, the price has to be very low for collaborative consumption to lead to lower ownership (per proposition 4). Since lower usage is only possible when there is lower ownership, the price threshold for lower usage must again be low.

Figure 3 illustrates the joint impact of p and c on ownership and usage. The price thresholds p_ω and p_q segment the full range of values of c and p into three regions, in which collaborative consumption leads to (i) lower ownership and lower usage, (ii) lower ownership but higher usage, and (iii) higher ownership and higher usage. These results highlight the fact that the impact of collaborative consumption on ownership and usage is perhaps more nuanced than what is sometimes claimed by advocates of collaborative consumption. The results could have implications for public policy. For example, in regions where the cost of ownership is high, the results imply that, unless rental prices are kept sufficiently low or the commission extracted by the platform is made sufficiently high, collaborative consumption would lead to more ownership and more usage. This could be an undesirable outcome if there are negative externalities associated with ownership and usage. Higher usage also implies less usage of the outside option (e.g., less use of public transport). However, the reverse could also be true. In particular, if rental prices are kept sufficiently low, collaborative consumption would lead to both lower ownership and lower usage.

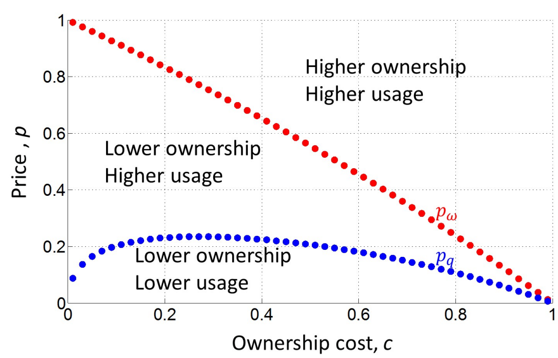


Figure 3: Ownership and usage for varying rental prices and ownership costs

Next, we examine the impact of collaborative consumption on consumer payoff. Consumer payoff is of course always higher with the introduction of collaborative consumption (consumers retain the option of either owning or not owning, but now enjoy the additional benefit of earning rental income if they decide to own or of fulfilling some of their usage through renting if they decide not to own). What is less clear is who, among consumers with different usage levels, benefit more from collaborative consumption.

Let $\pi^*(\xi)$ and $\hat{\pi}(\xi)$ denote respectively the consumer payoff with and without collaborative consumption. Then,

$$\hat{\pi}(\xi) = \begin{cases} 0 & \text{for} \quad 0 \le \xi < c; \\ \xi - c & \text{for} \quad c \le \xi \le 1, \end{cases}$$

and

$$\pi^*(\xi) = \begin{cases} (1-\alpha^*)\xi(1-p-d) & \text{for} \quad 0 \le \xi < \theta^*; \\ \xi + (1-\xi)\alpha^*[(1-\gamma)p - w] - c & \text{for} \quad \theta^* \le \xi \le 1. \end{cases}$$

Proposition 6. *Let* $\Delta(\xi) = \pi^*(\xi) - \hat{\pi}(\xi)$*. Then,*

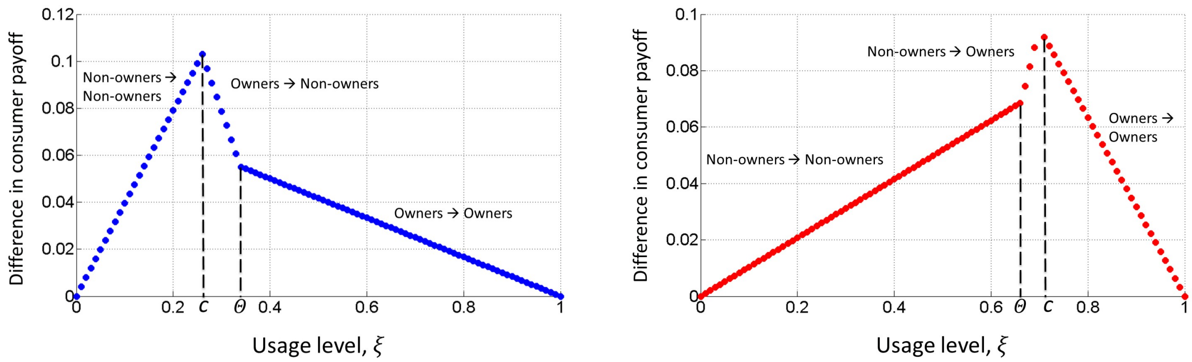
$$\Delta(\xi) = \begin{cases} (1-\alpha^*)\xi(1-p-d) & \textit{for} \quad 0 \le \xi < c; \\ -\alpha^*\xi - (1-\alpha^*)\xi(p+d) + c & \textit{for} \quad c \le \xi < \theta^*; \\ (1-\xi)\alpha^*[(1-\gamma)p - w] & \textit{for} \quad \theta^* \le \xi \le 1, \end{cases}$$

if $\theta^* \ge c$ *(or equivalently* $p \le p_\omega$*), and*

$$\Delta(\xi) = \begin{cases} (1-\alpha^*)\xi(1-p-d) & \textit{for} \quad 0 \le \xi < \theta^*; \\ \xi + (1-\xi)\alpha^*[(1-\gamma)p - w] - c & \textit{for} \quad \theta^* \le \xi < c; \\ (1-\xi)\alpha^*[(1-\gamma)p - w] & \textit{for} \quad c \le \xi \le 1, \end{cases}$$

if $\theta^* < c$ *(or equivalently* $p > p_\omega$*). In both cases, the difference in consumer payoff* $\Delta(\xi)$ *is positive, piecewise linear, strictly increasing on* $[0, c)$*, and strictly decreasing on* $[c, 1]$*.*

An important implication from Proposition 6 (from the fact that the difference in consumer surplus $\Delta(\xi)$ is strictly increasing on $[0, c)$ and strictly decreasing on $[c, 1]$) is that consumers who benefit the most from collaborative consumption are those who are indifferent between owning and not owning without collaborative consumption (recall that $[c, 1]$ corresponds to the population of owners in the absence of collaborative consumption). This can be explained by noting that there are always three segments of consumers (see Figure 4). In the case where $p \le p_w$, which corresponds to the case where ownership decreases with collaborative consumption, the first segment corresponds to consumers who are non-owners in the absence of collaborative consumption and continue to be non-owners with collaborative consumption (indicated by "non-owners→non-owners" in Figure 4). The benefit these consumers derive from collaborative consumption is due to fulfilling part of their usage through accessing a rented car. This benefit is increasing in their usage.



(a) $\theta^* > c$ (b) $\theta^* < c$

Figure 4: Impact of usage level on the difference in consumer payoff

The second segment corresponds to consumers who are owners in the absence of collaborative consumption and switch to being non-owners with collaborative consumption (indicated by "owners→non-owners"). These consumers have to give up the fulfillment of some usage (because a rental car may not always be available) and the amount they give up is increasing in their usage. Therefore, the amount of benefit they receive from renting decreases in their usage level. The third segment consists of consumers who are owners in the absence of collaborative consumption and continue to be owners with collaborative consumption (indicated by "owners→owners"). The benefit they experience is due to rental income. This income is decreasing in their usage (they have less capacity to rent when they have more usage). A similar explanation can be provided for the case where $p > p_w$.

5 The Platform's Problem

In this section, we consider the problem faced by the platform. A platform may decide, among others, on the price and commission fees. In this section, we focus on price as the primary decision made by the platform and treat other parameters as being exogenously specified[6]. There are of course settings where the price is a decision made by the owners or involves the renters (e.g., via a scheme where renters place bids and owners determine a winner). Owner-determined pricing

[6]A survey of major peer-to-peer car sharing platforms worldwide reveals that commission fees fall mostly within a relatively narrow range (from 30 to 40 percent for those that include insurance) and do not typically vary across markets in which platforms operates.

is observed when the products are heterogenous in their quality and features (e.g., Airbnb allows owners to set prices for their homes). Platform-determined pricing is plausible when the products are homogeneous (e.g., DriveMycar determines the rental price for each category of car; this is also observed in other peer-to-peer platforms such as Uber)[7]. In what follows, we provide analysis for two cases: a for-profit platform whose objective is to maximize revenue and a not-for-profit platform (e.g., a platform owned by a government agency or a municipality) whose objective is to maximize social welfare. Similar analysis could be carried out for other objectives or with additional constraints[8].

For a for-profit platform, the optimization problem can be stated as follows.

$$\max_p v_r(p) = \gamma p \alpha S(\theta), \tag{15}$$

$$\text{subject to } \pi_o(\theta) = \pi_r(\theta) \tag{16}$$

$$\alpha = \frac{D(\theta)}{D(\theta) + S(\theta),} \tag{17}$$

$$\pi_o(\xi) \geq \pi_r(\xi) \text{ for } \xi \geq \theta, \tag{18}$$

$$\pi_o(\xi) \leq \pi_r(\xi) \text{ for } 0 \leq \xi \leq \theta, \tag{19}$$

$$\pi_o(\xi) \geq 0 \text{ for } \xi \geq \theta, \tag{20}$$

$$\pi_r(\xi) \geq 0 \text{ for } 0 \leq \xi \leq \theta. \tag{21}$$

The constraints (16)-(17) are the defining equations for the equilibrium (θ^*, α^*). The incentive compatibility constraints (18)-(19) ensure that an individual who chooses to be an owner (renter) is better off being an owner (renter). The participation constraints (20)-(21) ensure that both owners and renters participate in collaborative consumption.

For a not-for-profit platform, the objective is to maximize social welfare (i.e., the sum of consumer surplus and platform revenue). Thus, the platform's problem can be stated as

[7] There are also hybrid settings where owners determine a minimum acceptable price but allow the platform to adjust it higher (e.g., RelayRides) or where the platform offers a suggested price (e.g., JustShareIt) but owners are allowed to deviate.

[8] For example, in settings where either ownership or usage are associated with negative externalities, the objective of a not-for-profit platform may be modified to include the social cost of these externalities. In settings where the platform is owned by the product manufacturer (an increasing number of manufacturers are setting up platforms to facilitate the peer-to-peer sharing of their products), the objective of the platform may account for product usage since higher usage leads to more frequent product replacements.

$$\max_p v_s(p) = \int_{[\theta,1]} (u(\xi) - (1-\xi)\alpha w - c) f(\xi) d\xi + \int_{[0,\theta)} (u(\beta\xi) - d\beta\xi) f(\xi) d\xi, \tag{22}$$

subject to constraints (16)-(21).

In the next three sections, we provide detailed analysis for the for-profit and not-for-profit platforms under the assumptions of Section 4. In Section 5.1 and 5.2, we consider the case where $(w, d) = (0, 0)$. In Section 5.3, we discuss the case where $(w, d) \neq (0, 0)$.

5.1 The For-Profit Platform

In this section, we assume $\gamma > 0$ (the platform's revenue is otherwise always zero). Under the assumptions of Section 4, the for-profit platform's problem can be restated as follows:

$$\max_p \quad v_r(p) = \tfrac{1}{2}\gamma p \alpha (1-\theta)^2, \tag{23}$$

subject to (10) and (11). It is difficult to analyze (23) directly. However, as the map between θ and p is strictly decreasing and continuously differentiable in both directions (by Lemma 2), we can use (10) and (11) to express p in terms of θ as

$$p(\theta) = \frac{-\theta^3 + 2c\theta^2 - 2c\theta + c}{\theta(\theta-1)(\gamma\theta-1)}, \tag{24}$$

and (23) as

$$\max_\theta \quad v_r(\theta) = \tfrac{\gamma}{2} \tfrac{(1-\theta)\theta(\theta^3-2c\theta^2+2c\theta-c)}{((1-\theta)^2+\theta^2)(\gamma\theta-1)} \quad \text{subject to } \theta \in [\underline{\theta}, \overline{\theta}] \tag{25}$$

where $\underline{\theta}$ is the solution to (10) and (11) at $p = 1$, $\overline{\theta}$ is the solution at $p = 0$, and $[\underline{\theta}, \overline{\theta}]$ is the set of solutions induced by $p \in [0, 1]$. We can use (24) to verify whether θ is in $[\underline{\theta}, \overline{\theta}]$. Specifically, $\theta < \underline{\theta}$ if $p(\theta) > 1$, $\theta \in [\underline{\theta}, \overline{\theta}]$ if $p(\theta) \in [0, 1]$, and $\theta > \overline{\theta}$ if $p(\theta) < 0$.

Proposition 7. *v_r is strictly quasiconcave in both p and θ.*

Proposition 7 shows that the platform's problem is not difficult to solve. Depending on the value of γ and c, v_r is either strictly decreasing or first strictly increasing then strictly decreasing on $[\underline{\theta}, \overline{\theta}]$. In both cases, the optimal solution to (25), which we denote by θ_r^*, is unique. We let p_r^*, ω_r^*, and

q_r^* denote the corresponding price, ownership, and usage, respectively. We also use the notation v_r^* to denote the optimal revenue $v_r(\theta_r^*)$.

Proposition 8. *The platform's optimal revenue, v_r^*, is strictly quasiconcave in c, first strictly increasing and then strictly decreasing.*

Proposition 8 suggests that a platform would be most profitable when the cost of ownership is "moderate" and away from the extremes of being either very high or very low. In these extreme cases, not enough transactions take place because of either not enough renters (when the cost of ownership is low) or not enough owners (when the cost of ownership is high). This result also implies that a platform may have an incentive to affect the cost of ownership. For example, when the cost of ownership is low, a platform may find it beneficial to impose a fixed membership fee on owners, increasing the effective cost of ownership. On the other hand, when the cost of ownership is high, the platform may find it beneficial to lower the effective cost of ownership by offering, for example, subsidies (or assistance with financing) toward the purchase of new products.

Proposition 9. *There exists a threshold $c_r \in (0, 1)$ such that optimal usage $\omega_r^* < \hat{\omega}$ if $c < c_r$, $\omega_r^* = \hat{\omega}_r$ if $c = c_r$, and $\omega_r^* > \hat{\omega}$ if $c > c_r$. Moreover, c_r is strictly increasing in γ.*

Proposition 9 shows that it continues to be possible, even when the price is chosen optimally by a revenue maximizing platform, for collaborative consumption to lead to either higher or lower ownership. In particular, collaborative consumption leads to higher ownership when the cost of ownership is sufficiently high (above the threshold c_r) and to lower ownership when the cost of ownership is sufficiently low (below the threshold c_r). This is easiest to understand in conjunction with Proposition 4. When the cost of ownership is high, the platform may need to charge a price p_r^* higher than p_ω to induce the optimal level of ownership. Similarly, when the cost of ownership is low, the platform may need to charge a price p_r^* lower than p_ω. By Proposition 4, $p_r^* > p_\omega$ and $p_r^* < p_\omega$ lead respectively to higher and lower ownership than that without collaborative consumption. It follows that usage can also be higher with collaborative consumption than without it. In particular, this is the case when ownership is higher (i.e., when $c > c_r$). When ownership is lower, usage is observed to be also mostly higher (except when c is very low). As illustrated in Figure 5, this is the case for the full range of values of γ.

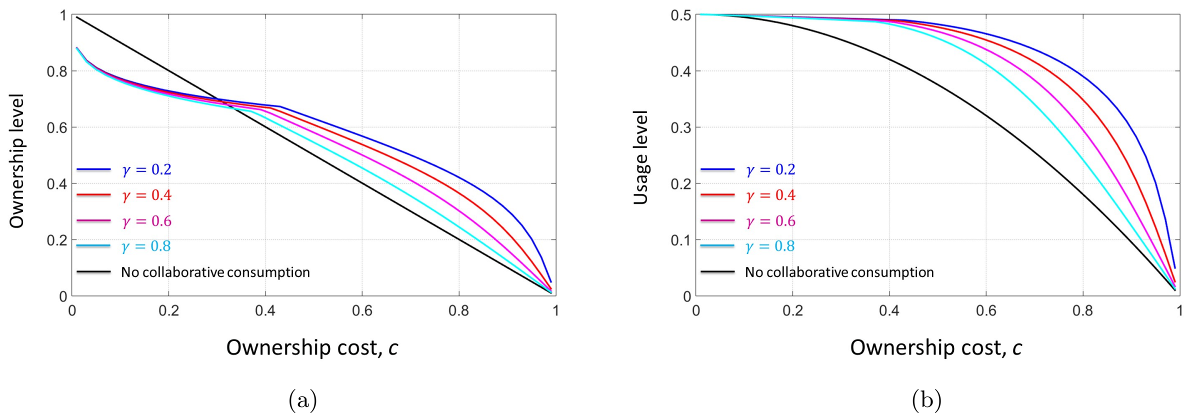


Figure 5: The impact of ownership cost on ownership and usage levels

5.2 The Not-for-Profit Platform

Analysis and results similar to those obtained for the for-profit platform can be obtained for the not-for-profit platform (i.e., a platform that maximizes social welfare). Under the assumptions of Section 4, the platform's problem can be restated as follows:

$$\max_p \quad v_s(p) = \tfrac{1}{2}(1 - \alpha\theta^2) - (1-\theta)c \tag{26}$$

subject to (10) and (11), or equivalently as

$$\max_\theta \quad v_s(\theta) = \tfrac{1}{2}\left(1 - \tfrac{\theta^4}{((1-\theta)^2+\theta^2)}\right) - c(1-\theta) \quad \text{subject to } \theta \in [\underline{\theta}, \overline{\theta}]. \tag{27}$$

In the following proposition, we show that social welfare, v_s, is concave in θ, indicating that computing the optimal solution for the not-for-profit platform is also not difficult.

Proposition 10. *v_s is strictly concave in θ, and strictly quasiconcave in p.*

Proposition 10 implies that (27) admits a unique optimal solution, which we denote by θ_s^*. We also use the notation v_s^* to denote the optimal social welfare $v_s(\theta_s^*)$. The following lemma characterizes θ_s^* for varying values of γ.

Lemma 11. *There exists a strictly positive decreasing function $\gamma_s(c)$ such that $\theta_s^* \in (\underline{\theta}, \overline{\theta})$ if $\gamma < \gamma_s$. Otherwise, $\theta_s^* = \underline{\theta}$.*

In other words, θ_s^* is an interior solution (satisfying $\frac{\partial v_s}{\partial \theta}(\theta_s^*) = 0$) if $\gamma < \gamma_s$. Otherwise, it is the boundary solution $\underline{\theta}$. In particular, θ_s^* never takes the value of $\overline{\theta}$. An important implication of Lemma 11 is that, when $\gamma < \gamma_s$, a not-for-profit platform that relies on price alone as a decision variable is able to achieve the maximum feasible social welfare realized under a *central planner* (a decision maker that can directly decide on θ).

Proposition 12. *If $\gamma \leq \gamma_s(c)$, then*

$$\max_{\theta \in [\underline{\theta}, \overline{\theta}]} v_s = \max_{\theta \in [0,1]} v_s.$$

Using Lemma 11, we can also show that social welfare under optimal pricing is strictly decreasing in the cost of ownership, which is perhaps consistent with intuition.

Proposition 13. *The optimal social welfare, v_s^*, is strictly decreasing in c.*

Similar to the case of for-profit platform, we can show that a not-for-profit platform can lead to either higher or lower level of ownership (relative to the case without collaborative consumption). Again, there is a threshold in the cost of ownership above which ownership is higher and below which ownership is lower. We omit the details for the sake of brevity.

In the remainder of this section, we compare outcomes under the for-profit and not-for-profit platforms. In the following proposition, we show that a not-for-profit platform would always charge a lower price than a for-profit platform. Therefore, it would also induce lower ownership and lower usage.

Proposition 14. *Let p_s^*, ω_s^* and q_s^* denote the optimal price, ownership and usage levels under a not-for-profit platform, respectively. Then, $p_s^* \leq p_r^*$, $\omega_s^* \leq \omega_r^*$, and $q_s^* \leq q_r^*$.*

In settings where there are negative externalities associated with ownership and usage, the result in Proposition 14 means that the not-for-profit platform also lowers such externalities. The fact that social welfare is maximized at prices lower than those that would be charged by a for-profit platform suggests that a regulator may be able to nudge a for-profit platform toward outcomes with higher social welfare by putting a cap on price.

Figure 6 illustrates the differences in social welfare between a system without collaborative consumption and systems with collaborative consumption under (a) a for-profit platform (a revenue-maximizing platform) and (b) a not-for-profit platform (social welfare-maximizing) platform. Systems with collaborative consumption can improve social welfare substantially, especially when the cost of ownership is neither too high nor too low (in those extreme cases, there are either mostly owners or mostly renters and, therefore, few transactions). However, the differences in social welfare between the for-profit and not-for-profit platforms are not very significant. This is because both platforms have a similar interest in maintaining a relative balance of renters and owners.

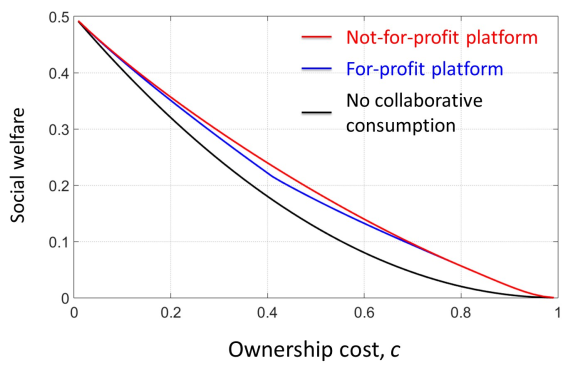
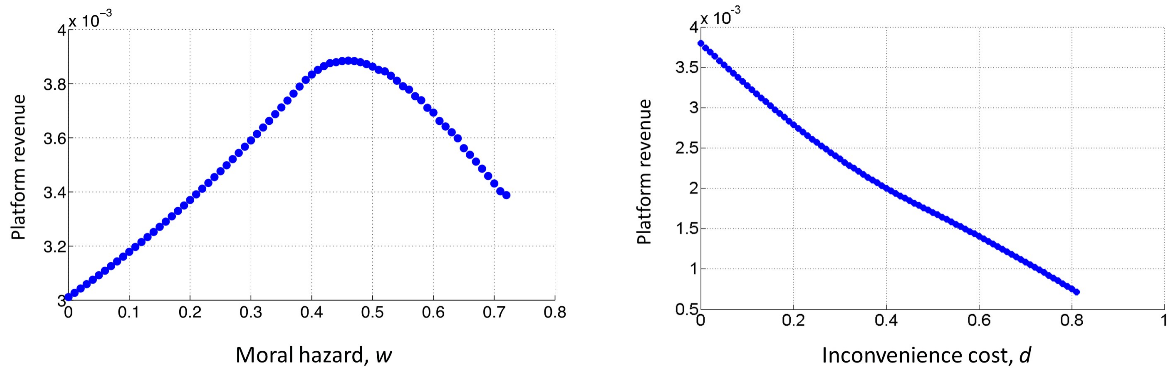


Figure 6: The impact of ownership cost on social welfare

5.3 The Impact of Moral hazard and Inconvenience Costs

In this section, we consider the case where $(w, d) \neq 0$. The moral hazard cost w reduces the payoff of owners and, therefore, places a lower bound on the set of admissible prices: $p \geq \frac{w}{(1-\gamma)}$. Similarly, the inconvenience cost d reduces the payoff of renters and, consequently, places an upper bound on the price: $p \leq 1 - d$. Obtaining analytical results is difficult. However, we are able to confirm numerically that all the results obtained for $(w, d) = 0$ continue to hold (details are omitted for brevity).

Of additional interest is the impact of w and d on platform revenue and social welfare. For both the for-profit and not-for-profit platforms, we observe that social welfare is decreasing in both w and d, regardless of the type of platform. This is consistent with intuition. However, revenue for the for-profit platform can be non-monotonic in w. In particular, when the cost of ownership is low, platform revenue can first increase then decrease with w. This effect appears related to the fact that platform revenue is non-monotonic in the cost of ownership. A higher value of w can be beneficial to the platform if it helps balance the amount of owners and renters, leading to a greater amount of transactions. An important implication from this result is that a for-profit platform may not have an incentive to eliminate all moral hazard. The inconvenience cost d does not have the same effect on platform revenue. An increase in d could lead to more transactions. However, it limits the price a platform could charge. The net effect is that the platform revenue is always decreasing in d. These effects are illustrated in Figure 7.



(a) $\gamma = 0.4$, $c = 0.2$, $d = 0$ (b) $\gamma = 0.4$, $c = 0.8$, $w = 0$

Figure 7: Platform revenue for varying moral hazard and inconvenience costs

6 Concluding Comments

In this paper, we described an equilibrium model of collaborative consumption. We characterized equilibrium outcomes, including ownership and usage levels, consumer surplus, and social welfare. We compared each outcome in systems with and without collaborative consumption and examined the impact of various problem parameters including rental price, platform's commission fee, cost of ownership, owner's moral hazard cost, and renter's inconvenience cost. Our findings indicate that, depending on the rental price, collaborative consumption can result in either lower or higher ownership and usage levels, with higher ownership and usage levels more likely when the cost of ownership is high. We showed that consumers always benefit from collaborative consumption, with individuals who, in the absence of collaborative consumption, are indifferent between owning and not owning benefitting the most. We also showed that the platform's profit is not monotonic in the cost of ownership, implying that a platform is least profitable when the cost of ownership is either very high or very low (also suggesting that a platform may have an incentive in affecting the cost of ownership by, for example, imposing membership fees or providing subsidies). In addition, we observed that platform profit can be non-monotonic in the moral hazard cost, suggesting that a for-profit platform may not have the incentive to eliminate all moral hazard.

Possible avenues for future research are many. We mention few examples. It would be useful to consider settings where individuals are heterogeneous along multiple dimensions (e.g., usage level, sensitivity to inconvenience, and sensitivity to moral hazard). It would also be useful to examine settings where there is competition among multiple platforms, with owners and renters having the option of participating in one or more such platforms. As we mentioned in Section 4, it would be of interest to analyze outcomes under platforms that may have alternative objectives, such as a not-for-profit platform that may be interested in minimizing negative externalities associated with ownership and usage or a for-profit platform operated by the product manufacturer that may be interested in accounting for usage.

7 Acknowledgements

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Appendix

Proofs

Proof of Theorem 1: The right hand side of equation (11) is strictly increasing in θ. The right hand side of equation (10) is decreasing in α as

$$\frac{\partial\theta}{\partial\alpha} = \frac{(c-p-d)((1-\gamma)p-w)-c(1-p-d)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \leq 0.$$

The above inequality holds because, on one hand, $(c-p-d)((1-\gamma)p-w)-c(1-p-d) = (1-p-d)((1-\gamma)p-w-c)-(1-c)((1-\gamma)p-w) \leq 0$ if $(1-\gamma)p-w \leq c$, and on the other hand, $(c-p-d)((1-\gamma)p-w)-c(1-p-d) = (c-p-d)((1-\gamma)p-w-c)-c(1-c) < 0$ if $(1-\gamma)p-w > c$.

Solving equation (10) with respect to α, we obtain

$$\alpha = \frac{c-(p+d)\theta}{\theta(1-p-d)+(1-\theta)((1-\gamma)p-w)}.$$

Define $g(\theta) := \frac{c-(p+d)}{\theta(1-p-d)+(1-\theta)((1-\gamma)p-w)}$ and $h(\theta) := \frac{\theta^2}{(1-\theta)^2+\theta^2}$. So $g(\theta)-h(\theta)$ decreases with θ. If $\theta=0$, then $g(\theta) = \frac{c}{(1-\gamma)p-w} > 0$, $h(\theta)=0$, and $g(0)-h(0)>0$. If $\theta=1$, then $g(\theta) = \frac{c-(p+d)}{1-p-d}$, $h(\theta)=1$, and $g(1)-h(1) = \frac{c-1}{1-p-d} < 0$. Therefore, by the Intermediate Value Theorem, there exists a unique θ^* such that $g(\theta^*)=h(\theta^*)$. This θ^* along with the corresponding α^* given by (11) is the unique pair of (θ,α) satisfying both (10) and (11).

Proof of Lemma 2: Let

$$\begin{aligned} f(\theta,\alpha,p,\gamma,c,w,d) &= (f_1(\theta,\alpha,p,\gamma,c,w,d), f_2(\theta,\alpha,p,\gamma,c,w,d)) \\ &= (\theta - \frac{c-((1-\gamma)p-w)\alpha}{p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha}, \alpha - \frac{\theta^2}{(1-\theta)^2+\theta^2}), \end{aligned}$$

and

$$g(p,\gamma,c,w,d) = (g_1(p,\gamma,c,w,d), g_2(p,\gamma,c,w,d))$$
$$= (\theta^*(p,\gamma,c,w,d), \alpha^*(p,\gamma,c,w,d)).$$

By Theorem 1, g is the unique solution to $f(\theta,\alpha)=0$.

We first show (θ^*,α^*) is continuous on Ω. Observe f is continuous on $[0,1]^2\times\Omega$ unless $d+(p-((1-\gamma)p-w)\alpha)+(1-p-d)\alpha=0$, or equivalently, unless $(\alpha,p,d)=(0,0,0)$ or $(\alpha,p,\gamma,w,d)=(1,1,0,0,0)$. Take any sequence in $y_n\to y_0=(p_0,\gamma_0,c_0,w_0,d_0)\in\Omega$. For any subsequence y_{n_k}, as $(0,1)^2$ is relatively compact, there exists a subsubsequence $y_{n_{k_l}}$ such that $g(y_{n_{k_l}})\to(\theta_0,\alpha_0)\in[0,1]^2$. If $(p_0,d_0)=(0,0)$, then $\alpha_0\neq 0$, for in this case $f_1(g(y_{n_{k_l}}),y_{n_{k_l}})\to-\infty\neq 0$. Similarly, if $(p_0,\gamma_0,w_0,d_0)=(1,0,0,0)$, then $\alpha_0\neq 1$, for in this case, $f_1(g(y_{n_{k_l}}),y_{n_{k_l}})\to\infty\neq 0$. As f is continuous everywhere else, we have $f(\theta_0,\alpha_0,y_0)=\lim f(g(y_{n_{k_l}}),y_{n_{k_l}})=0$. By Theorem 1, (θ_0,α_0) is the unique solution to $f(\theta_0,\alpha_0,y_0)=0$ in $(0,1)^2$. So, $g(y_0)=(\theta_0,\alpha_0)$. As $g(y_0)$ is independent of the choice of y_{n_k}, by the subsequence principle, $g(y_n)\to g(y_0)=(\theta_0,\alpha_0)$, whence g is continuous.

To show g is continuously differentiable on Ω°, we use Euler's notation D for differential operators. In particular, for any component x and y of f,

$$D_x f = \begin{pmatrix} \frac{\partial f_1}{\partial x} \\ \frac{\partial f_2}{\partial x} \end{pmatrix},$$

and

$$D_{(x,y)} f = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix}.$$

It follows that

$$D_{(\theta,\alpha)} f = \begin{pmatrix} 1 & \frac{c(1-p-d)-(c-p-d)((1-\gamma)p-w)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \\ -\frac{2\theta(1-\theta)}{[(1-\theta)^2+\theta^2]^2} & 1 \end{pmatrix}$$

As $\frac{c(1-p-d)-(c-p-d)((1-\gamma)p-w)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2}>0$ on Ω° (see proof of Theorem 1), $D_{(\theta,\alpha)}f$ is invertible. By the

Implicit Function Theorem, g is continuously differentiable, and for each component x,

$$D_x g = -[D_{(\theta,\alpha)} f]^{-1} D_x f, \tag{28}$$

where

$$[D_{(\theta,\alpha)} f]^{-1} = \frac{1}{\det(D_{(\theta,\alpha)} f)} \begin{pmatrix} 1 & \frac{(c-p-d)((1-\gamma)p-w)-c(1-p-d)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \\ \frac{2\theta(1-\theta)}{[(1-\theta)^2+\theta^2]^2} & 1 \end{pmatrix}.$$

Calculating $D_x f$ for each component x leads to

$$D_p f = \begin{pmatrix} \frac{(1-\gamma)\alpha(\alpha-c)+c(1-\alpha)+(1-\gamma)\alpha(1-\alpha)d+\alpha(1-\alpha)w}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \\ 0 \end{pmatrix},$$

$$D_\gamma f = \begin{pmatrix} \frac{p\alpha(c-p-d-(1-p-d)\alpha)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \\ 0 \end{pmatrix},$$

$$D_c f = \begin{pmatrix} -\frac{1}{p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha} \\ 0 \end{pmatrix},$$

$$D_w f = \begin{pmatrix} \frac{\alpha(c-p-d-(1-p-d)\alpha)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \\ 0 \end{pmatrix},$$

and

$$D_d f = \begin{pmatrix} \frac{(1-\alpha)(c-((1-\gamma)p-w)\alpha)}{(p+d+(1-p-d)\alpha-((1-\gamma)p-w)\alpha)^2} \\ 0 \end{pmatrix}.$$

First, $D_c f_1 < 0$ is obvious. Second, from Equation (10), $0 < \theta < 1$ implies $c - ((1-\gamma)p - w)\alpha > 0$ and $c - p - d - (1-p-d)\alpha < 0$. So, we have $D_d f_1 > 0$, $D_\gamma f_1 < 0$ and $D_w f_1 < 0$. Third, $(1-\gamma)\alpha(\alpha-c)+c(1-\alpha) > 0$ if $\alpha > c$, and $(1-\gamma)\alpha(\alpha-c)+c(1-\alpha) = ((1-\gamma)\alpha-c)(\alpha-c)+c(1-c) > 0$ if $\alpha \leq c$ imply $D_p f_1 > 0$. Therefore, from (28), we conclude $\frac{\partial\theta^*}{\partial p} < 0$, $\frac{\partial\alpha^*}{\partial p} < 0$, $\frac{\partial\theta^*}{\partial\gamma} > 0$, $\frac{\partial\alpha^*}{\partial\gamma} > 0$, $\frac{\partial\theta^*}{\partial c} > 0$, $\frac{\partial\alpha^*}{\partial c} > 0$, $\frac{\partial\theta^*}{\partial w} > 0$, $\frac{\partial\alpha^*}{\partial w} > 0$, $\frac{\partial\theta^*}{\partial d} < 0$, and $\frac{\partial\alpha^*}{\partial d} < 0$.

Proof of Proposition 3: By Lemma 2, $\omega^*(p,\gamma,c,w,d)$ and $q^*(p,\gamma,c,w,d)$, as continuously differentiable functions, are strictly decreasing in γ, c, and w, and strictly increasing in p and d.

Proof of Proposition 4: By Equations (10) and (11), θ^* is the unique solution to

$$\phi(\theta,p)=[(1+p+d)-((1-\gamma)p-w)]\theta^3-[2(p+d+c)-((1-\gamma)p-w)]\theta^2+(p+d+2c)\theta-c=0 \quad (29)$$

in $(0,1)$. Replacing θ by c in (29) leads to

$$\phi(c,p)=c(1-c)[(1-\gamma c)p-((1-d)(1-c)+wc)]=0.$$

So,

$$p_\omega=\frac{(1-d)(1-c)+wc}{1-\gamma c}$$

is the rental price that induces $\theta^*=c$, or equivalently, $\omega^*=1-\theta^*=1-c=\hat{\omega}$. The assumption $\frac{w}{1-\gamma}<1-d$ ensures that $\frac{w}{1-\gamma}<p_\omega<1-d$. As ω^* is strictly increasing in p, the first statement follows. In addition,

$$\frac{\partial p_\omega}{\partial \gamma}=\frac{c((1-d)(1-c)+wc)}{(1-\gamma c)^2}>0,$$

$$\frac{\partial p_\omega}{\partial c}=\frac{w-(1-\gamma)(1-d)}{(1-\gamma c)^2}<0,$$

$$\frac{\partial p_\omega}{\partial w}=\frac{c}{1-\gamma c}>0,$$

and

$$\frac{\partial p_\omega}{\partial d}=\frac{c-1}{1-\gamma c}<0.$$

Proof of Proposition 5: From Proposition 4, we have

$$q^*(p_\omega)=\frac{1-\alpha^*c^2}{2}>\frac{1-c^2}{2}=\hat{q}.$$

As q^* is continuously increasing in p, we know that either (i) or (ii) is true. Moreover, let $\underline{p}=\frac{w}{1-\gamma}$ be the minimal admissible price. Then, (i) is true if and only if $q^*(\underline{p})<\hat{q}$. Otherwise, (ii) is true. In the rest of the proof, we show that both (i) and (ii) can be true, depending on whether

$\frac{w}{1-\gamma} + d < t$ or $\frac{w}{1-\gamma} + d \geq t$, for some $t \in (0,1)$.

To this end, we first claim that $q^*(\underline{p})$ is strictly increasing in $\frac{w}{1-\gamma} + d$. It suffices to show that $\theta^*(\underline{p})$ is strictly decreasing in $\frac{w}{1-\gamma} + d$, as q^* is a strictly decreasing function of θ^*. From (29), we have

$$\begin{aligned}\phi(\theta, \underline{p}) =& (1 + \frac{w}{1-\gamma} + d)\theta^3 - 2(\frac{w}{1-\gamma} + d + c)\theta^2 + (\frac{w}{1-\gamma} + d + 2c)\theta - c \\ =& \theta^3 - 2c\theta^2 + 2c\theta - c + (\frac{w}{1-\gamma} + d)(\theta^3 - 2\theta^2 + \theta).\end{aligned}$$

It is easy to verify that $\phi(\theta, \underline{p})$ is strictly increasing in $\frac{w}{1-\gamma} + d$. Therefore, $\theta^*(\underline{p})$, the unique solution to $\phi(\theta, \underline{p}) = 0$, must be strictly decreasing in $\frac{w}{1-\gamma} + d$.

Next, we claim that $q^*(\underline{p}) > \hat{q}$ if $\frac{w}{1-\gamma} + d = 1$ (this corresponds to when $\frac{w}{1-\gamma} + d$ is at its largest), and $q^*(\underline{p}) < \hat{q}$ if $\frac{w}{1-\gamma} + d = 0$ (this corresponds to when $\frac{w}{1-\gamma} + d$ is at its smallest). Substituting (11) into (13) yields

$$q^*(p) = \frac{1}{2} - \frac{\theta^{*4}}{2(2\theta^{*2} - 2\theta^* + 1)}.$$

So, $q^*(p) < \hat{q}$ if and only if

$$\psi(p) = \theta^{*4} - 2c^2\theta^{*2} + 2c^2\theta^* - c^2 > 0.$$

Note that

$$\begin{aligned}\psi(\underline{p}) =& \psi(\underline{p}) - c\phi(\theta^*(\underline{p}), \underline{p}) \\ =& \theta^{*4} - c\theta^{*3} - c(\frac{w}{1-\gamma} + d)(\theta^{*3} - 2\theta^{*2} + \theta^*).\end{aligned}$$

Then, if $\frac{w}{1-\gamma} + d = 1$, we have

$$\begin{aligned}\psi(\underline{p}) =& \theta^*(\theta^{*3} - 2c\theta^{*2} + 2c\theta^* - c) \\ <& \theta^*\phi(\theta^*(\underline{p}), \underline{p}) \\ =& 0,\end{aligned}$$

and if $\frac{w}{1-\gamma} + d = 0$, we have (by Proposition 4),

$$\begin{aligned}\psi(\underline{p}) =& \theta^{*4} - c\theta^{*3} \\ >& 0.\end{aligned}$$

Finally, as $q^*(\underline{p})$ is strictly increasing in $\frac{w}{1-\gamma} + d$, by the Intermediate Value Theorem, there exists $t \in (0, 1)$ such that $q^*(\underline{p}) < \hat{q}$ if $\frac{w}{1-\gamma} + d < t$ and $q^*(\underline{p}) \geq \hat{q}$ if $\frac{w}{1-\gamma} + d \geq t$. This completes the proof.

Proof of Proposition 6: The derivation of $\Delta(\xi)$ is straightforward. It is easy to see that $\Delta(\xi)$ is piecewise linear, increasing on $[0, c)$, and decreasing on $[c, 1]$. The fact that $\Delta(\xi)$ is positive follows from the incentive compatibility and participation constraints (18)-(21).

Proof of Proposition 7: We show v_r is strictly quasiconcave in θ. Then, the statement for p will follow as a consequence, for the composition of a quasiconcave function and a monotone function is still quasiconcave. To maximize total revenue, the platform has to balance rental price p and the amount of successful transaction $\alpha(1-\theta)^2/2$. Among the duo, rental price p is a decreasing function in θ, while the amount of transaction $\alpha(1-\theta)^2/2$ is a quasiconcave function in θ that peaks at $\theta = \frac{1}{2}$, the point where rental supply equals to rental demand. This implies $v_r(\theta)$ in (25) is strictly decreasing on $[\max\{\frac{1}{2}, \underline{\theta}\}, \overline{\theta}]$. Therefore, to show $v_r(\theta)$ is strictly quasiconcave on $[\underline{\theta}, \overline{\theta}]$, it suffices to show it is strictly quasiconcave on $(0, \frac{1}{2})$.

As v_r is smooth in θ, we have

$$\frac{\partial v_r}{\partial \theta} = \frac{\gamma}{2} \frac{c(2\theta^2 - 2\theta + 1)^2(\gamma\theta^2 - 2\theta + 1) + \theta^3(-4\gamma\theta^4 + (8\gamma + 6)\theta^3 - (8\gamma + 12)\theta^2 + (3\gamma + 11)\theta - 4)}{(\gamma\theta - 1)^2(2\theta^2 - 2\theta + 1)^2} \tag{30}$$

Note that $\frac{\partial v_r}{\partial \theta}(0) > 0$ and $\frac{\partial v_r}{\partial \theta}(\frac{1}{2}) < 0$. We claim that the numerator is strictly decreasing on $\theta \in (0, \frac{1}{2})$. Suppose this is true. Then $\frac{\partial v_r}{\partial \theta}$ is first positive then negative on $(0, \frac{1}{2})$, whence v_r is strictly quasiconcave on $(0, \frac{1}{2})$. To see the claim is true, set

$$g_1(\theta, \gamma) = (2\theta^2 - 2\theta + 1)^2(\gamma\theta^2 - 2\theta + 1),$$

and

$$g_2(\theta,\gamma) = \theta^3(-4\gamma\theta^4 + (8\gamma+6)\theta^3 - (8\gamma+12)\theta^2 + (3\gamma+11)\theta - 4).$$

We show that g_1 and g_2 are both strictly decreasing in θ.

First, we show g_1 is strictly decreasing.

$$\frac{\partial g_1}{\partial \theta} = 2(2\theta^2 - 2\theta + 1)(6\gamma\theta^3 - (4\gamma+10)\theta^2 + (\gamma+10)\theta - 3).$$

It suffices to show that

$$h_1(\theta,\gamma) = 6\gamma\theta^3 - (4\gamma+10)\theta^2 + (\gamma+10)\theta - 3 < 0.$$

Indeed, we have

$$\frac{\partial h_1}{\partial \theta} = 18\gamma\theta^2 - (8\gamma+20)\theta + (\gamma+10),$$

and

$$\frac{\partial^2 h_1}{\partial\gamma\partial\theta} = 18\theta^2 - 8\theta + 1.$$

$\frac{\partial^2 h_1}{\partial\gamma\partial\theta} > 0$ implies $(\frac{\partial h_1}{\partial\theta}(\cdot,\gamma) : \gamma \in (0,1))$ is pointwise bounded below by $\frac{\partial h_1}{\partial\theta} h_1(\cdot,0)$. As $\frac{\partial h_1}{\partial\theta}(\theta,0) = -20\theta + 10 > 0$ on $(0,\frac{1}{2})$, $\frac{\partial h_1}{\partial\theta}(\cdot,\gamma) > 0$ for all γ. So, we conclude $h_1(\cdot,\gamma)$ is increasing. Then, $h_1(\frac{1}{2},\gamma) = -\frac{1}{2} + \frac{\gamma}{4} < 0$ implies $h_1(\theta;\gamma) < 0$. Therefore, g_1 is strictly decreasing.

Second, we show g_2 is strictly decreasing.

$$\frac{\partial g_2}{\partial \theta} = -4\theta^2(7\gamma\theta^4 - (12\gamma+9)\theta^3 + (10\gamma+15)\theta^2 - (3\gamma+11)\theta + 3).$$

It suffices to show that

$$h_2(\theta,\gamma) = 7\gamma\theta^4 - (12\gamma+9)\theta^3 + (10\gamma+15)\theta^2 - (3\gamma+11)\theta + 3 > 0.$$

We have

$$\frac{\partial h_2}{\partial \theta} = 28\gamma\theta^3 - (36\gamma+27)\theta^2 + (20\gamma+30)\theta - (3\gamma+11),$$

$$\frac{\partial^2 h_2}{\partial \theta^2} = 84\gamma\theta^2 - (72\gamma+54)\theta + (20\gamma+30),$$

and

$$\frac{\partial^3 h_2}{\partial\gamma\partial\theta^2} = 84\theta^2 - 72\theta + 20.$$

$\frac{\partial^3 h_2}{\partial\gamma\partial\theta^2} > 0$ implies $(\frac{\partial^2 h_2}{\partial\theta^2}(\cdot,\gamma) : \gamma \in (0,1))$ is pointwise bounded below by $\frac{\partial^2 h_2}{\partial\theta^2}(\cdot,0)$. As $\frac{\partial^2 h_2}{\partial\theta^2}(\theta,0) = -54\theta + 30 > 0$ on $(0,\frac{1}{2})$, $\frac{\partial^2 h_2}{\partial\theta^2}h_2 > 0$. It follows that $\frac{\partial h_2}{\partial\theta}$ is increasing in θ. Then, $\frac{\partial h_2}{\partial\theta}(\frac{1}{2},\gamma) = -\frac{11}{4} + \frac{3}{2}\gamma < 0$ implies $\frac{\partial h_2}{\partial\theta} < 0$. So, $h_2(\cdot,\gamma)$ is decreasing. At last, we have $h_2(\frac{1}{2},\gamma) = \frac{1}{8} - \frac{1}{16}\gamma > 0$, whence $h_2 > 0$. Therefore, g_2 is strictly decreasing.

Proof of Proposition 8: We first show that θ_r^* is increasing in c. Observe $\theta_r^* = \underline{\theta}$ if $\underline{\theta} \geq \frac{1}{2}$, and $\theta_r^* \leq \frac{1}{2}$ if $\underline{\theta} < \frac{1}{2}$. As $\underline{\theta} = \theta^*(1)$ is increasing in c, it suffices to assume $\underline{\theta} < \frac{1}{2}$ and show θ_r^* is increasing in c in this case. By (25),

$$\frac{\partial^2 \pi_p}{\partial\theta\partial c} = \frac{\gamma(\gamma\theta^2 - 2\theta + 1)}{(\gamma\theta - 1)^2} > 0$$

for $\theta < \frac{1}{2}$. This implies v_r is supermodular in (θ, c) on $[0,\frac{1}{2}]^2$. As the correspondence $c \to [\underline{\theta},\frac{1}{2}]$ is increasing, by the Topkis Theorem (Topkis [1998, Lemma 2.8.1]), θ_r^* is increasing in c.

Next, we show that v_r^* is strictly quasiconcave in c. As θ in Lemma 2 is continuously differentiable, $\frac{\partial v_r}{\partial c}$ are continuous. By the Envelope Theorem (Milgron and Segal [2002, Corollary 4]),

$$\frac{\partial v_r^*}{\partial c}(\gamma, c) = \frac{\partial v_r}{\partial c}(p_r^*, \gamma, c)$$

on any compact interval in $(0,1)$. As $(0,1)$ can be covered by an increasing union of compact subintervals, these envelope equations hold on entire $(0,1)$. Therefore,

$$\frac{\partial v_r^*}{\partial c}(\gamma, c) = \gamma p_r^* \frac{2\theta_r^*(\theta_r^* - 1)(2\theta_r^* - 1)(\theta_r^{*2} - \theta_r^* + 1)}{(2\theta_r^{*2} - 2\theta_r^* + 1)^2} \frac{\partial\theta}{\partial c}(p_r^*, \gamma, c).$$

Lemma 2 shows $\frac{\partial\theta}{\partial c} > 0$. So, $\frac{\partial v_r^*}{\partial c} > 0$ if $\theta_r^* < \frac{1}{2}$, $\frac{\partial v_r^*}{\partial c} = 0$ if $\theta_r^* = \frac{1}{2}$, and $\frac{\partial v_r^*}{\partial c} < 0$ if $\theta_r^* > \frac{1}{2}$. As θ_r^* is increasing in c, we conclude v_r^* is quasiconcave in c. Moreover, substituting θ with $\frac{1}{2}$ from (30) yields $\frac{\partial v_r}{\partial\theta}(\frac{1}{2}) = \frac{\gamma(2c\gamma+\gamma-3)}{4(\gamma-2)^2} < 0$. This implies $\theta_r^* = \frac{1}{2}$ iff $\underline{\theta} = \frac{1}{2}$. Therefore, v_r^* is strictly quasiconcave in c.

It remains to be shown that v_r^* has a strictly increasing as well as a strictly decreasing segment. Given what we have shown, it is sufficient to prove that, as c ranges through $(0,1)$, $\overline{\theta} = \theta(0)$ has a segment below $\frac{1}{2}$, and $\underline{\theta} = \theta(1)$ has a segment above $\frac{1}{2}$. From (29), $\theta(0) < \frac{1}{2}$ iff $\phi(\frac{1}{2}, 0) > 0$, and

$\theta(1) > \frac{1}{2}$ iff $\phi(\frac{1}{2}, 1) < 0$. The former is equivalent to

$$c < \frac{1}{4},$$

and the latter to

$$c > \frac{3}{4} - \frac{1}{4}\gamma.$$

Therefore, v_r^* is strictly increasing when $c < \frac{1}{4}$, and strictly decreasing when $c > \frac{3}{4} - \frac{1}{4}\gamma$. As c ranges through $(0, 1)$, both segments are non-empty.

Proof of Proposition 9: By Proposition 4, $c \in (\underline{\theta}, \overline{\theta})$ for $(\gamma, c) \in (0, 1)^2$. As v_r is quasiconcave, $\omega_p^*(c) \leq \hat{\omega}(c)$ iff $\theta_r^*(c) \geq c$ iff $\frac{\partial v_r}{\partial \theta}(c) \geq 0$. Replacing θ by c in (30), we have

$$\frac{\partial v_r}{\partial \theta}(c) = \frac{\gamma}{2} \frac{c(1-c)(2c^4 - 6c^3 + (7+\gamma)c^2 - 5c + 1)}{(\gamma c - 1)^2(2c^2 - 2c + 1)^2}.$$

Let

$$g(c, \gamma) = 2c^4 - 6c^3 + (7+\gamma)c^2 - 5c + 1.$$

Then $\frac{\partial v_r}{\partial \theta}(c) \geq 0$ iff $g(c, \gamma) \geq 0$. Moreover, $g(0, \gamma) = 1 > 0$, $g(1, \gamma) = \gamma - 1 < 0$, and g is strictly convex in c, as $\frac{\partial^2 g}{\partial c^2} = 24c^2 - 36c + 14 + 2\gamma > 0$. It follows that $g(c, \gamma) = 0$ has a unique solution $c_r(\gamma) \in (0, 1)$. We have proved the first statement. For the second statement, observe $g(c, \gamma_1) < g(c, \gamma_2)$ if $\gamma_1 < \gamma_2$. As $g(c_r(\gamma_1), \gamma_1) = 0$, we have $g(c_r(\gamma_1), \gamma_2) > 0$, and by the first statement, it must be true that $g(c, \gamma_2) > 0$ for $c < c_r(\gamma_1)$. Therefore, we must have $c_r(\gamma_2) > c_r(\gamma_1)$.

Proof of Proposition 10: It suffices to show that v_s is strictly concave in θ. As θ is strictly decreasing in p, this implies that v_s is strictly quasiconcave in p. From (27), we have

$$\frac{\partial^2 v_s}{\partial \theta^2} = -\frac{2\theta^2(1-\theta)^2(2\theta^2 - 2\theta + 3)}{(2\theta^2 - 2\theta + 1)^3} < 0$$

on $(0, 1)$. Therefore, v_s is strictly concave on $[\underline{\theta}, \overline{\theta}]$.

Proof of Lemma 11 We denote by, θ_c^*, the unique optimal solution to $\max_{\theta \in [0,1]} v_s$ for $c \in [0, 1]$.

By the Maximum Theorem and Topkis Theorem, θ_c^* is continuously increasing in c. For $c \in (0,1)$, as

$$\frac{\partial v_s}{\partial \theta} = \frac{1}{4}(-1 + 4c - 2\theta + \frac{(1-2\theta)}{(2\theta - 2\theta + 1)^2}) \tag{31}$$

is stricly positive when $\theta = 0$, and strictly negative when $\theta = 1$, we know that v_s, apart from being concave, is first increasing, then decreasing on $[0,1]$. Therefore, $\frac{\partial v_s}{\partial \theta}(\theta_c^*) = 0$. It follows that θ_c^* satisfies $\frac{(2\theta_c^{*5} - 3\theta_c^{*4} + 2\theta_c^{*3})}{(2\theta_c^{*2} - 2\theta_c^* + 1)^2} = c$. Replacing c by $\frac{(2\theta_c^{*5} - 3\theta_c^{*4} + 2\theta_c^{*3})}{(2\theta_c^{*2} - 2\theta_c^* + 1)^2}$ in (24) yields

$$p(\theta_c^*) = \frac{\theta_c^{*2}}{(1 - \gamma\theta_c^*)(2\theta_c^{*2} - 2\theta_c^* + 1)} > 0.$$

If $p(\theta_c^*) < 1$, then $\theta_c^* \in (\underline{\theta}, \overline{\theta})$, in which case, $\theta_s^* = \theta_c^*$. Else, if $p(\theta^*) \geq 1$, then $\theta_c^* \leq \underline{\theta}$, in which case, $\theta_s^* = \underline{\theta}$.

Observe $p(\theta_c^*) < 1$ iff

$$\gamma \leq \frac{(\theta_c^* - 1)^2}{\theta_c^*(2\theta_c^{*2} - 2\theta_c^* + 1)}.$$

So, if we set $\gamma_s = \frac{(\theta_c^* - 1)^2}{\theta_c^*(2\theta_c^{*2} - 2\theta_c^* + 1)}$, then $\theta_c^* \in (\underline{\theta}, \overline{\theta})$ if $\gamma < \gamma_s$, and $\theta_c^* \leq \underline{\theta}$ if $\gamma \geq \gamma_s$. It is easy to see that γ_s is decreasing in θ_c^*, whence it is decreasing in c.

Proof of Proposition 12: As v_s is strictly concave on $[0,1]$, Lemma 11 implies θ_s^* is socially optimal if $\gamma \leq \gamma_s(c)$.

Proof of Proposition 13: To prove the proposition, we introduce the Lagrangian function

$$L(\theta, \mu, \gamma, c) = v_s(\theta) + \mu_1(\theta - \underline{\theta}) + \mu_2(\overline{\theta} - \theta).$$

Equation (31) shows that $\frac{\partial v_s}{\partial \theta}$ is bounded on $(\theta, \gamma, c) \in [0,1] \times [0,1) \times (0,1)$. By the first order condition

$$\begin{aligned}
\frac{\partial v_s}{\partial \theta}(\theta_s^*) + \mu_1^* - \mu_2^* = 0; \\
\mu_1^*(\theta_s^* - \underline{\theta}) = 0; \\
\mu_2^*(\overline{\theta} - \theta_s^*) = 0; \\
\mu_1^*, \mu_2^* \geq 0,
\end{aligned}$$

the Kuhn-Tucker vector μ^* is unique and bounded by $\sup |\frac{\partial v_s}{\partial \theta}|$ (at most one of μ_1^*, μ_2^* is non-zero). By the Saddle-point Theorem, (θ_s^*, μ^*) is a pair of optimal solution and Kuhn-Tucker vector iff it is a saddle point of the Lagrangian. It follows that there is a unique saddle point (θ_s^*, μ^*) for each (γ, c). As $\frac{\partial L}{\partial c}$ is continuous, the Envelope Theorem for saddle-points (Milgron and Segal [2002, Theorem 4 and 5]) implies that

$$\begin{aligned}
\frac{\partial v_s^*}{\partial c}(c) &= \frac{\partial L}{\partial c}(\theta_s^*, \mu^*, c) \\
&= -(1 - \theta_s^*) - \mu_1^* \frac{\partial \underline{\theta}}{\partial c} + \mu_2^* \frac{\partial \overline{\theta}}{\partial c}
\end{aligned}$$

holds everywhere (In general, the envelope equation holds almost everywhere without unique saddle-points). By Lemma 11, μ_2^* is always 0, and by Lemma 2, $\frac{\partial \underline{\theta}}{\partial c} > 0$. So, we have $\frac{\partial v_s^*}{\partial c} < 0$ everywhere. Therefore, v_s^* is strictly decreasing in c.

Proof of Proposition 14: Let

$$g(\theta, c) = -\frac{\theta(\theta^3 - 2c\theta^2 + 2c\theta - c)}{((1-\theta)^2 + \theta^2)}.$$

Then, $v_r(\theta, c) = \frac{\gamma(1-\theta)}{2(1-\gamma\theta)} g(\theta, c)$. Let θ_r be the unique solution to

$$\frac{\partial v_r}{\partial \theta} = \frac{\gamma(\gamma - 1)}{2(1-\gamma\theta)^2} g(\theta, c) + \frac{\gamma(1-\theta)}{2(1-\gamma\theta)} D_\theta g(\theta, c) = 0$$

in $(0, 1/2)$, and θ_s be the unique solution to

$$\frac{\partial v_s}{\partial \theta} = \frac{1}{4}(-1 - 2\theta + \frac{(1-2\theta)}{(2\theta^2 - 2\theta + 1)^2} + 4c) = 0$$

in $(0, 1)$. We claim that $\theta_r \le \theta_s$. Suppose the claim is true. Then, $\theta_r^* = \underline{\theta} \le \theta_s^*$ whenever $\theta_r < \underline{\theta}$, $\theta_r^* = \theta_r \le \theta_s = \theta_s^*$ whenever $\theta_r, \theta_s \in [\underline{\theta}, \overline{\theta}]$, and $\theta_r^* \le \overline{\theta} = \theta_s^*$ whenever $\theta_s > \overline{\theta}$. In all the cases, we have $\theta_r^* \le \theta_s^*$.

To prove the claim, observe g is strictly concave in θ on $[0, 1]$, since

$$\frac{\partial^2 g}{\partial \theta^2} = -\frac{4\theta^2(1-\theta)^2(2\theta^2 - 2\theta + 3)}{(2\theta^2 - 2\theta + 1)^3} < 0.$$

Let θ_g be the unique solution to

$$\frac{\partial g}{\partial \theta} = \frac{1}{2}(-1 - 2\theta + \frac{1-2\theta}{(2\theta^2 - 2\theta + 1)^2} + 2c) = 0$$

in $(0, 1)$. We show that $\theta_r \le \theta_g \le \theta_s$.

To see that $\theta_r \le \theta_g$, note that

$$\frac{\partial v_r}{\partial \theta}(\theta_g) = \frac{\gamma(\gamma - 1)}{2(1 - \gamma\theta_g)^2} g(\theta_g, c) \le 0.$$

As v_r is strictly quasiconcave on $[0, 1/2]$, it must be true that $\theta_r \le \theta_g$. To see that $\theta_g \le \theta_s$, note that

$$h(\theta) = -1 - 2\theta + \frac{1-2\theta}{(2\theta^2 - 2\theta + 1)^2}$$

is strictly decreasing. So, it must be true that $\theta_g \le \theta_s$.